Global uniqueness in General Relativity: the Strong Cosmic Censorship Conjecture.

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The Einstein field equations
(as seen in Uyuni, Bolivia; © Madalena Miranda 2005)

General Relativity (GR) in a nutshell

GR is the study of Lorentzian manifolds (M, g), i.e.,

g is a pseudo-Riemannian metric
with signature $(-, +, +, +)$,

satisfying the Einstein Field Equations (1915):

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \ .$$

- $R_{\mu\nu}$ =Ricci curvature, $R =$ scalar curvature $\left[\sim \partial^2 g \right]$
- Λ is the cosmological constant,
- $T_{\mu\nu} =$ energy-momentum tensor.
  It describes the matter content of spacetime.

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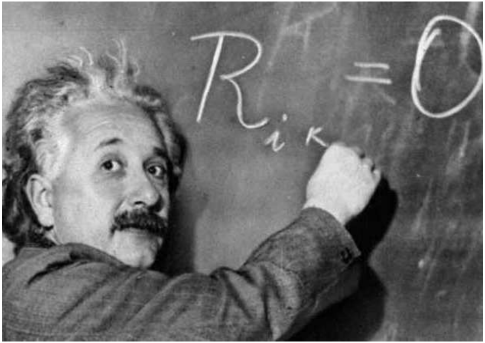
John Wheeler:

"Matter tells space how to curve,
and space tells matter how to move"

Minkowski spacetime (1907).



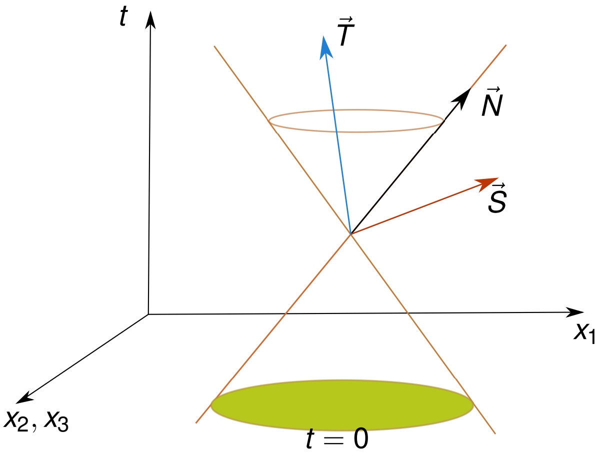
The simplest vacuum solution with $\Lambda = 0$ is given by:

- $M = \mathbb{R}^4$
- $g = \mathrm{diag}(-1, 1, 1, 1)$

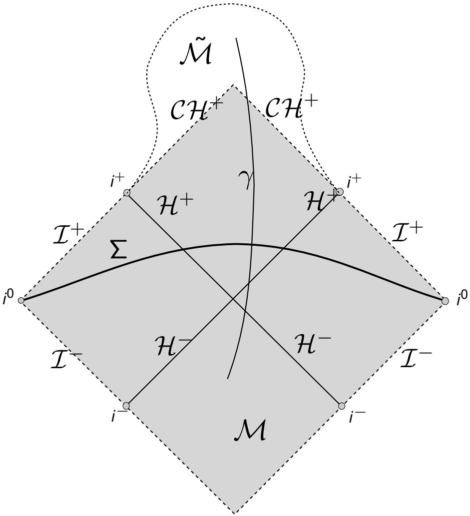
The metric is usually presented in the form

$$ds^2 = -dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \ .$$

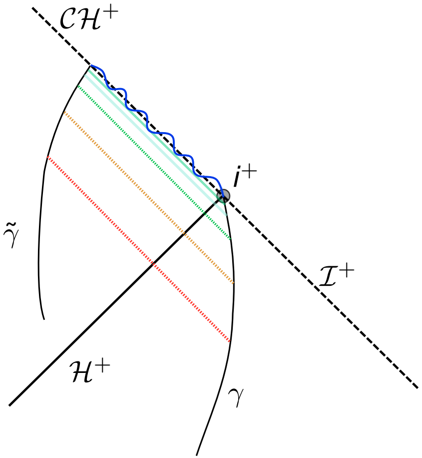
Minkowski and causality



The Reissner-Nordström (1918) and Kerr (1963) black holes



The blue-shift effect. Penrose (1960s).



The Strong Cosmic Censorship Conjecture

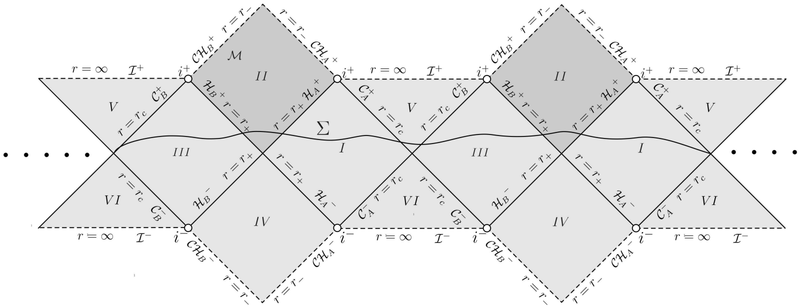
The maximal development of generic "appropriate" initial data for "reasonable" Einstein-matter systems, is future inextendible as a "suitably regular" Lorentzian manifold.

- "suitably regular"= continuous metric with locally squared integrable connection coefficients ($\Gamma \in L^2$).
- Relevant partial results (by Christodoulou, Dafermos and Luk) suggest that this conjecture holds for asymptotically flat ($\Lambda = 0$) black holes.

Adding a positive cosmological constant $\Lambda > 0$.

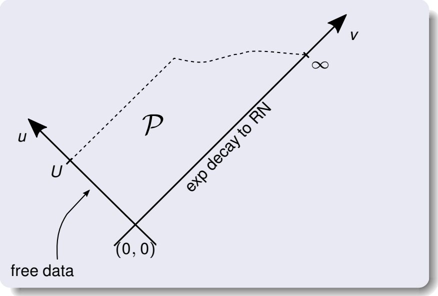
- The cosmological constant Λ was introduced by Einstein in 1917 to construct a static cosmological solution.
- Shortly after, it was rejected by Einstein himself (as his biggest blunder(?)) after the astronomical observations of Edwin Hubble.
- After 1998, with the observation of the accelerated expansion of the Universe, an enormous consensus as grown concerning the existence of a small but positive cosmological constant in our Universe!

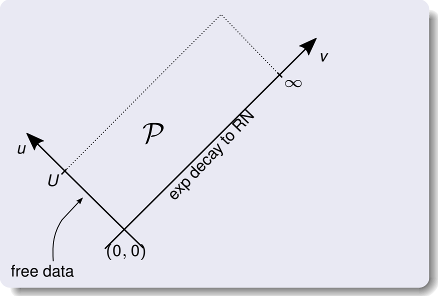
Reissner-Nordström-de Sitter.



A problem we've been working on.
joint with: Pedro Girão, José Natário and Jorge D. Silva

For the spherically symmetric Einstein-Maxwell-scalar field system with a cosmological constant consider the IVP:





- We find large classes of (close to extremal) data for which the metric extends with $\Gamma \in L^2$!
- Is this a counter example for the Strong Cosmic Censorship Conjecture, in the presence of positive cosmological constant?
- Not exactly! To obtain a counter-example we still have to:
  - show that that the data profile we are considering (cosmological Price's Law) appears generically in black hole formation
  - remove the spherical symmetry assumption.

In Reissner-Nordström-de Sitter, solutions to the wave equation

$$\Box_g \phi = 0$$

with smooth and spherically symmetric initial data, satisfy (in Eddington-Finkelstein coordinates) the following estimate along the event horizon:

$$\left| \partial_v \phi_{|\mathcal{H}^+} \right| \leq C_p e^{-pv}$$

for any $p < \min\{\kappa_+, \kappa_c\}$.

Beyond spherical symmetry
joint with: Anne Franzen.

In Reissner-Norsdtröm-de Sitter, consider a solution of the wave equation

$$\Box_g \phi = 0$$

decaying as before but without any symmetry assumptions on the initial data.
If

$$2 \min\{p, \kappa_+\} > \kappa_-$$

then ϕ is bounded and as bounded local energy, up to the Cauchy horizon.

What's next:

- Show that Price's law holds for the (non-linear) spherically symmetric Einstein-Maxwell-scalar field system with a positive cosmological constant.
- Extend the results with Franzen to the Kerr metric.
- Attack the full problem: the Einstein equations with no symmetry assumptions.

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